

Title: How High (Patterns and Functions)

Brief Overview:

Students will complete investigations to discover patterns and develop equations for linear models. Students will use manipulatives to gain knowledge of real-life applications of algebraic concepts. Students will construct scatter plots, as well as use them to analyze and interpret data.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Number and Operations

Students will demonstrate their ability to find relationships among numbers.

Algebra

Students will demonstrate their ability to understand patterns, relations, and functions. They will demonstrate their ability to represent and analyze mathematical situations and structures using algebraic symbols. They will demonstrate their ability to represent and understand quantitative relationships and analyze change in various contexts.

Measurement

Students will demonstrate their ability to apply appropriate techniques and tools to determine measurements.

Data Analysis and Probability

Students will demonstrate their ability to select and use appropriate statistical methods to analyze data.

Grade/Level:

Grades 6-8

Duration/Length:

This unit will take approximately four 50 minute periods (including the performance assessment).

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Defining equations or expressions for multiplication/division and addition/subtraction linear patterns
- Constructing scatter plots
- Measuring using a metric and standard ruler
- Using variables appropriately

Student Outcomes:

Students will:

- discover and define linear patterns using manipulatives.
- create data tables and develop equations for linear patterns.
- construct and interpret scatter plots of linear models.

Materials/Resources/Printed Materials:

- Activity sheets (included)
- Rainbow cubes or interlocking cubes (for each student)
- Dot paper (two sheets per student)
- Rulers (one for each student)
- Meter sticks (one for each pair of students)
- Various sizes of styrofoam cups with lips (at least four cups of two different sizes for each pair of students)
- Stackable chairs (two or three for each group of students)

Development/Procedures:**Day 1**

Materials needed: Activity #1 "Patterns in the Real World: Tiling Your Bathroom Floor"

Rainbow cubes or interlocking cubes (for each student)

Dot paper (two sheets per student)

Rulers (one per student)

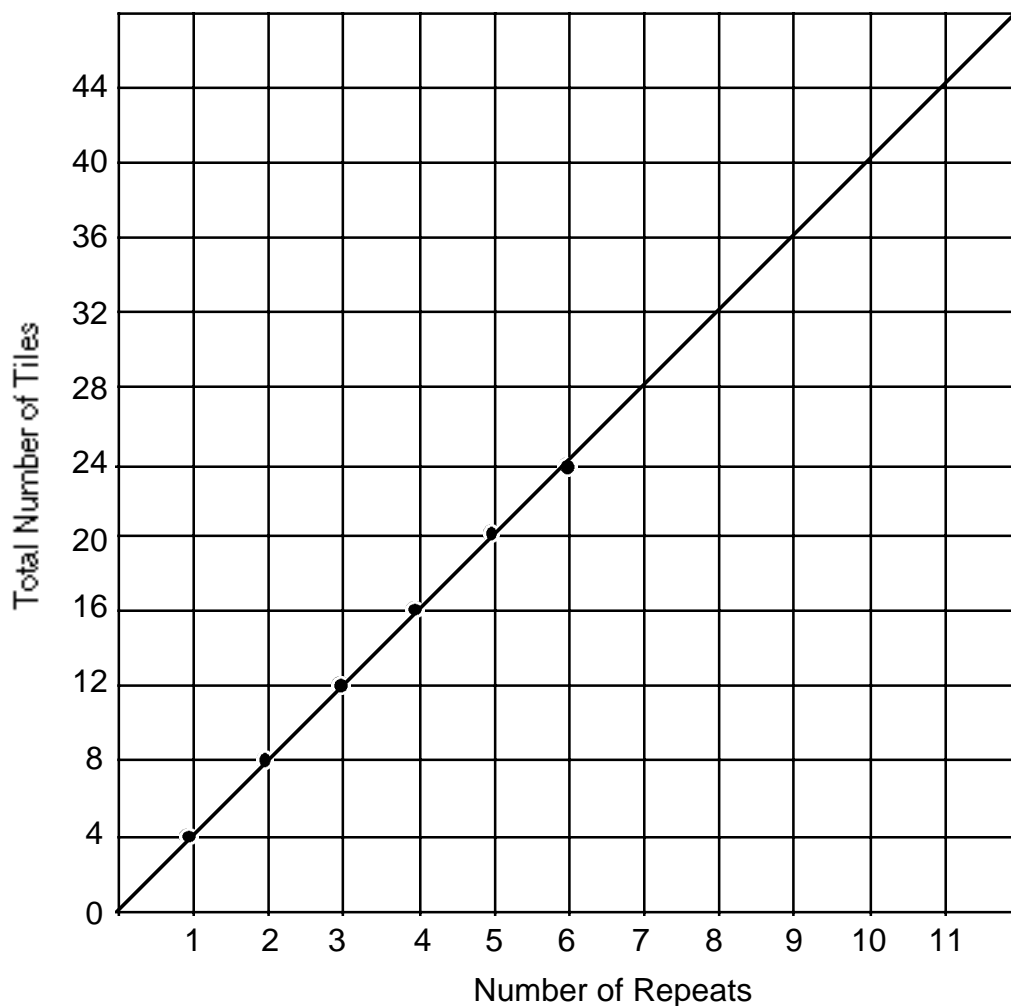
Students will work individually to complete Activity #1. Students will complete two linear patterns, make scatter plots to represent the data, and make predictions about the data using their table or graph. Students will use rainbow cubes or interlocking cubes and dot paper to help them visualize the patterns.

Scoring Key for Activity #1 - "Patterns in the Real World: Tiling Your Bathroom Floor"

Part A

1. 1 point- correctly answers with 4 tiles
0 points- incorrectly answers
2. 1 point-correctly answers with 8 tiles
0 points- incorrectly answers
3. 1 point- correctly answers with 12 tiles
0 points- incorrectly answers
4. 2 points- correctly completes chart
1 point- one or two mistakes but mostly correct
0 points- more than 3 mistakes
5. 3 points- correctly creates an appropriate graph with data from chart, using an appropriate title, scale, and labels
2 points- correctly creates an appropriate graph with a few mistakes but overall complete
1 point- correctly creates a graph with major mistakes
0 points- incorrectly creates a graph OR does not complete a graph

Tiles Needed to Tile a Floor



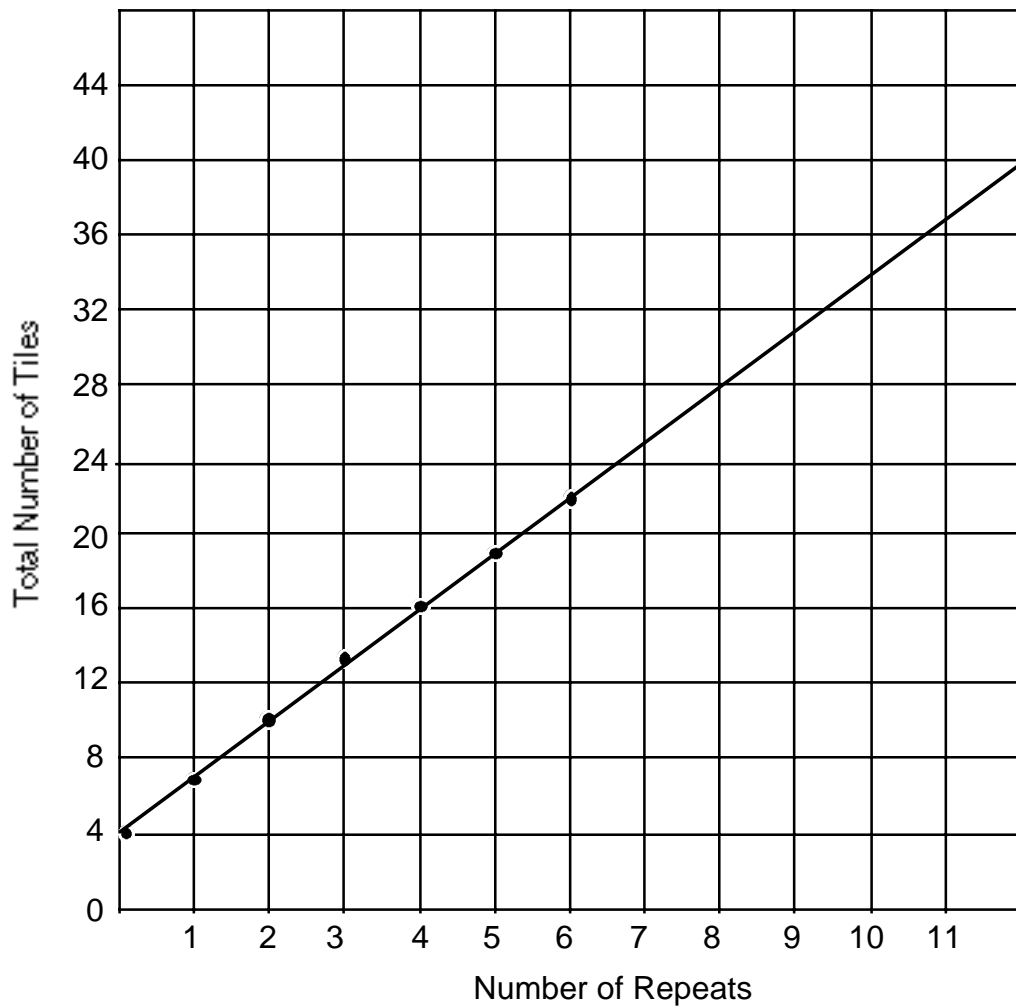
6. 2 points- correctly and clearly describes that the total number of tiles is four times the amount of the number of repeats
1 point- correctly describes the relationship with some confusion or problems
0 points- incorrectly describes the relationship or does not describe the relationship
7. 2 points- correctly predicts that 10 repeats would take 40 tiles because each repeat takes 4 tiles and there are 10 repeats so 10 times 4 is 40; also 20 repeats would take 80 tiles since each repeat takes 4 tiles and there are 20 repeats so 20 times 4 is 80
1 point- correctly predicts how many tiles are needed for 10 and 20 repeats but without explanation OR
1 point- correctly predicts how many tiles are needed for either 10 OR 20 repeats with explanation
0 points- incorrectly predicts predicts both OR incomplete
8. 1 points-correct equation $y = 4x$
0 points- completely incorrect or no equation
9. 1 point- correctly identifies that you would have 50 repeats with 200 tiles
0 points- incorrectly identifies the answer

Part B

1. 2 points- correctly completes chart
1 point- one or two mistakes but mostly correct
0 points- more than two mistakes
2. 3 points- correctly creates an appropriate graph with data from chart, using an appropriate title, scale, and labels
2 points- correctly creates an appropriate graph with a few mistakes but overall complete
1 point- correctly creates a graph with major mistakes
0 points- incorrectly creates a graph or does not complete a graph
3. 1 point- correctly answers that they started with 4 tiles
0 points- incorrectly answers the starting amount
4. 1 point- correctly answers that 3 tiles are needed for each repeat
0 points- incorrectly answers the amount needed for a repeat
5. 2 points- correctly predicts that 10 repeats would take 34 tiles with mathematical justification
1 point- correctly predicts how many tiles are needed but does not include an explanation
0 points- incorrectly predicts the number of tiles needed
6. 1 point- correctly answers that 79 tiles are needed for the 25th repeat
0 points- incorrectly answers the question

7. 1 points-correct equation $y=3x + 4$
0 points- completely incorrect or no equation
8. 1 point- correctly answers that 200 tiles would make 66 repeats
(does not include original 4 tiles)
0 points- incorrectly answers the question

Tiles Needed to Tile a Floor



Day 2/Day 3

Materials needed: **Activity #2 "Stacking Up"**
 Rulers (one per student)
 Various sizes of styrofoam cups with lips (at least six cups of
 two different sizes for each pair of students)

Students will work in pairs to complete Activity #2. Each pair of students will define linear patterns using at least two different sizes of styrofoam cups. Students will have at least four cups of two different sizes. Students will measure the height of the base cup (which is the constant). Then students will add one cup to make a stack of two cups and measure the height from the bottom of the base cup to the top of the second cup. Students will then add a second cup to make a stack of three cups and measure the height from the base cup to the top of the third cup. Students will then add a third cup to make a stack of four cups and measure the height from the base cup to the top of the fourth cup. Students will use this information to create a table of values, construct a scatter plot to represent the data, and develop an equation for the data. Note: the height of the base cup is the constant and the height of the lip of each cup is the coefficient of x .

Scoring Key for Activity #2 - "Stacking Up"

Part A

1. 2 points - chart is correctly completed
 1 point - chart has one or two mistakes
 0 points - chart has more than two mistakes
2. Not scored
3. 2 points - both correct with units
 1 point - one correct with units or both correct without units
 0 points - anything else
4. 3 points - correctly creates an appropriate graph with data from chart, using an
 appropriate title, scale, and labels
 2 points - correctly creates an appropriate graph with one or two errors
 1 point - correctly creates an appropriate graph with more than two errors
 0 points - incorrectly creates a graph or does not complete a graph
5. 1 point- indicates as the number of cups increases, the height also increases
 0 points - anything else
6. Not scored
7. 1 point - indicates the repeat of the pattern used multiplication
 0 points - anything else
8. 2 points - constant, has appropriate explanation
 1 point - constant, has incorrect or no explanation
 0 points - anything else
9. 1 point - correct equation
 0 points - incorrect response

Part B

1. 2 points - chart is correctly completed
1 point - chart has one or two mistakes
0 points- chart has more than two mistakes
2. 3 points - correctly creates an appropriate graph with data from chart, using an appropriate title, scale, and labels
2 points - correctly creates an appropriate graph with one or two errors
1 point - correctly creates an appropriate graph with more than two errors
0 points - incorrectly creates a graph or does not complete a graph
3. 1 point- correct equation
0 points - incorrect response
4. 2 points - uses a mathematical approach, student indicates the measure of the cup will determine the height of the stack
1 point- provides a reasonable explanation to determine the number of cups, but does not include a mathematical approach
0 points - anything else

Performance Assessment: Day Four

Materials needed: "Clean Up" Performance Assessment Activity
Rulers (one per student)
Meter sticks (one for each group of students)
Stackable chairs (two or three for each group of students)

Students will determine how chairs can be stacked so that only a small portion of the floor is used. Students will explain how they determined the height of the stack of chairs and the number of stacks they will need. They will also determine how many stacks will be needed if there are x number of chairs. Students will construct a scatter plot to represent the number of chairs added and the height of the chairs.

Scoring Rubric:

Part A

not scored

Part B

1. not scored
2. not scored
3. not scored
4. 2 points - Table is complete and accurate.
1 point - Table has one error.
0 points - Table has more than one error or is incomplete.

5. 1 point - Correct equation.
0 points - Incorrect equation.
6. 3 points - Correctly creates an appropriate graph with data from chart, using an appropriate title, scale, and labels.
2 points - Correctly creates an appropriate graph with a few mistakes but overall complete.
1 point - Correctly creates a graph with major mistakes.
0 points - Incorrectly creates a graph OR does not complete a graph.
7. not scored
8. not scored
9. 2 Question is completely answered and justified with mathematical reasoning.
1 Question is answered but has no mathematical reasoning.
0 Question is incorrect or incomplete.

Extension/Follow Up:

- Students can construct scatter plots using graphing calculators.
- Students can find linear regression models using graphing calculators.
- Students can create their own linear patterns and have other students find the equations that fit the patterns.

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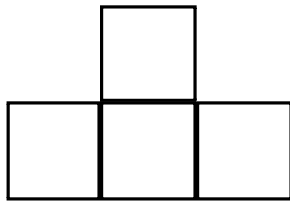
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Activity #1

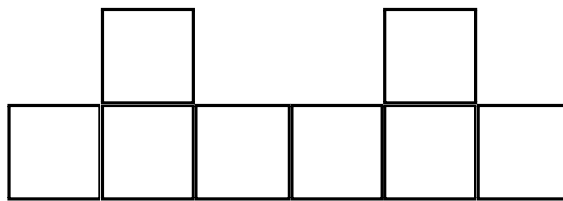
Patterns in the Real World: *Tiling Your Bathroom Floor*

Part A

Your parents have asked you to tile their bathroom floor. When tiling a bathroom floor, you need to have a design for the border. This pattern will repeat all the way around the bathroom. Below is the pattern your parents have chosen to be repeated around the bathroom:



*Basic Design
(One Repeat)*



Two Repeats

1. How many tiles are needed for the first repeat?

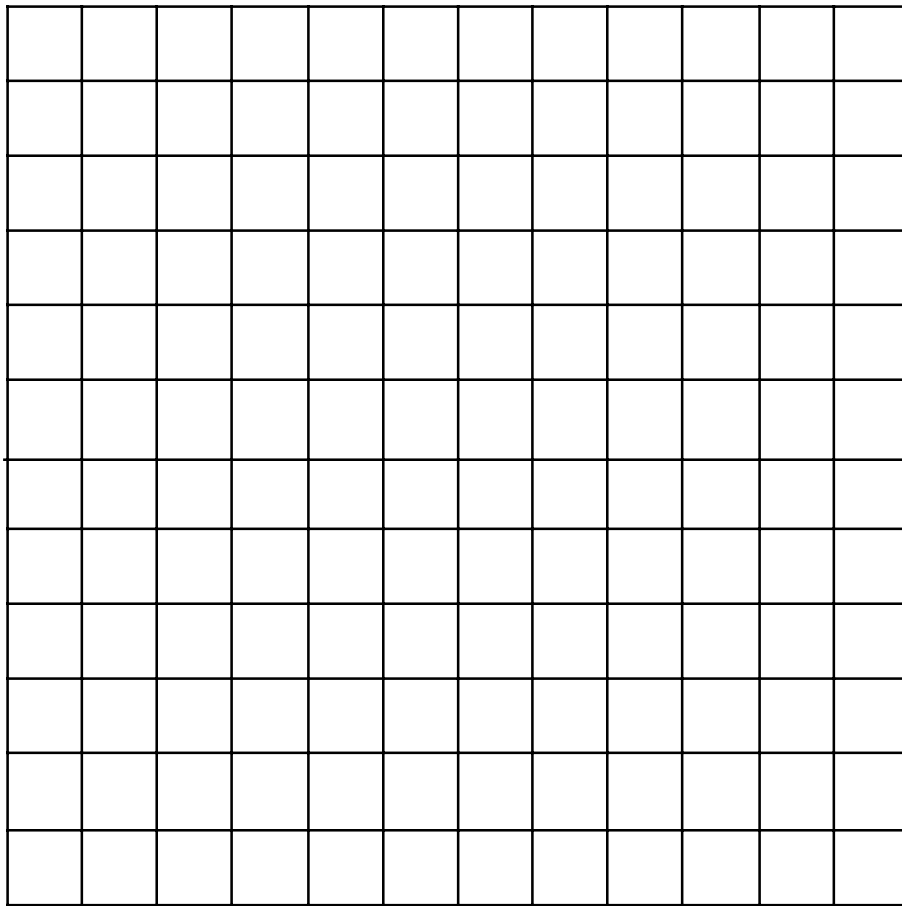
2. How many for the second repeat?

3. How many for the third repeat?

4. On the provided dot paper, draw how the tile border will appear after six repeats of the pattern. Then complete the table below.

# of Repeats	Total # of Tiles
0	0
1	4
2	8
3	
4	
5	
6	

5. Using the grid below, display the data in a scatter plot.



6. Describe how the total number of tiles is related to the number of repeats.

7. Predict and explain how many tiles would be needed to make 10 repeats? _____

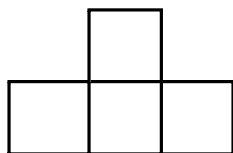
20 repeats? _____

8. If x is the number of repeats and y is the total number of tiles, write an equation to predict the number of tiles needed?

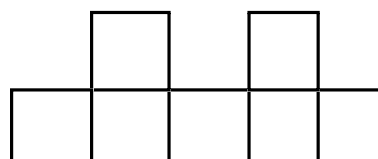
9. If you have 200 tiles, how many repeats are possible?

Part B

After you started cementing the tiles to the floor, you realize you won't have enough for the original pattern. The original design is glued down and can't be removed. You decide to change the pattern so you don't run out of tiles. Your new design is displayed below.



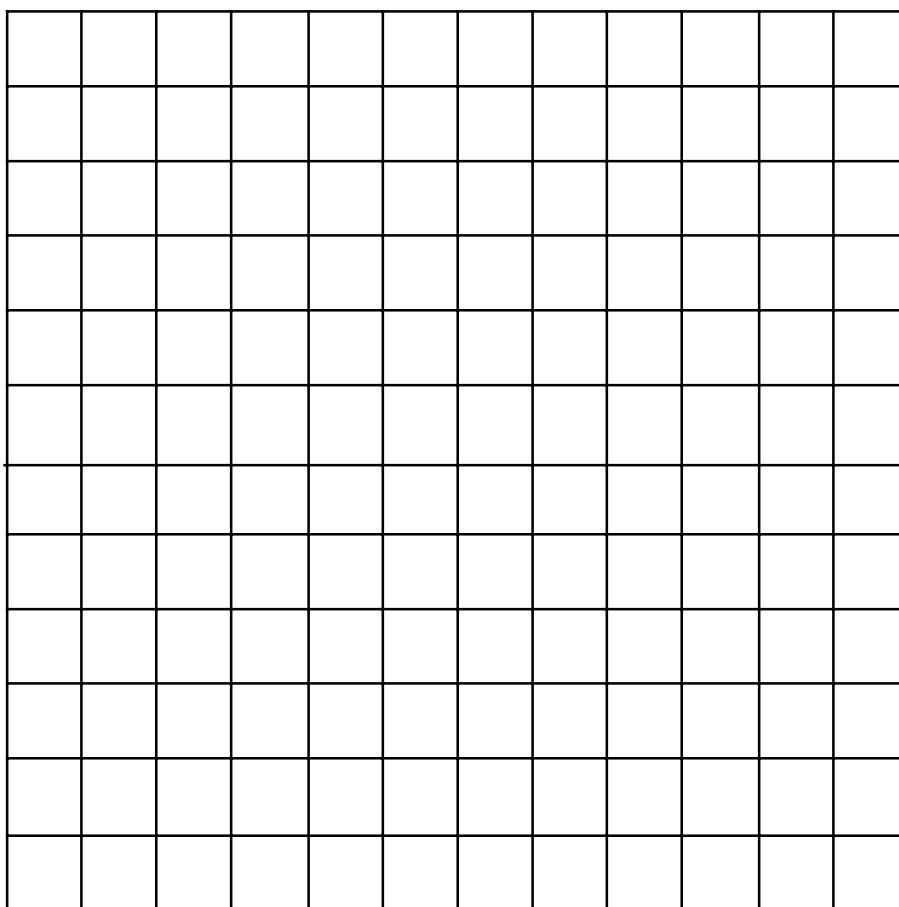
Original design



New Design
(first repeat)

1. On the dot paper provided, complete this pattern for 6 repeats and complete this table.
2. Using the grid below, display the data in a scatter plot.

# of Repeats	Total # of Tiles
0	4
1	
2	
3	
4	
5	
6	



3. How many tiles did you start with originally? _____

4. How many tiles are needed for each repeat? _____

5. Use the table and graph to predict and explain the number of tiles for the tenth repeat. _____

6. How many tiles are needed for the 25th repeat? _____

7. If x is the number of repeats and y is the total number of tiles, write an equation for this relationship.

Hint:

Total number of tiles	=	starting number for original design	+	the number for each repeat times the number of repeats
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8. If you have 200 tiles not glued to the floor, how many repeats are possible?

Activity #2

Stacking Up

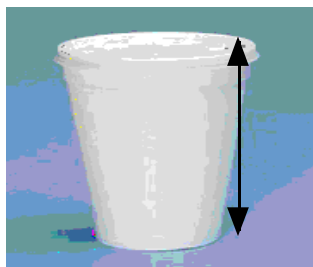
Your family owns a small coffee shop downtown. You work there on weekends to earn money for a new mountain bike. One of your jobs every Saturday morning is to stock the shelves. This includes unpacking the big boxes of styrofoam cups and putting them on the shelves. Because the shelves are above your head you often stack the cups too high, the top of the stack hits the ceiling and they fall to the ground. One morning you realize you can avoid this problem if you knew how high to make each stack of cups so it will fit on the shelf perfectly.

The following investigation will help you find the perfect stack.

PART A

With your partner and the stack of five small cups complete the following steps.

1. Find the height of one cup to the nearest tenth of a cm. **This is the initial height.** Record this measure on the data table provided.

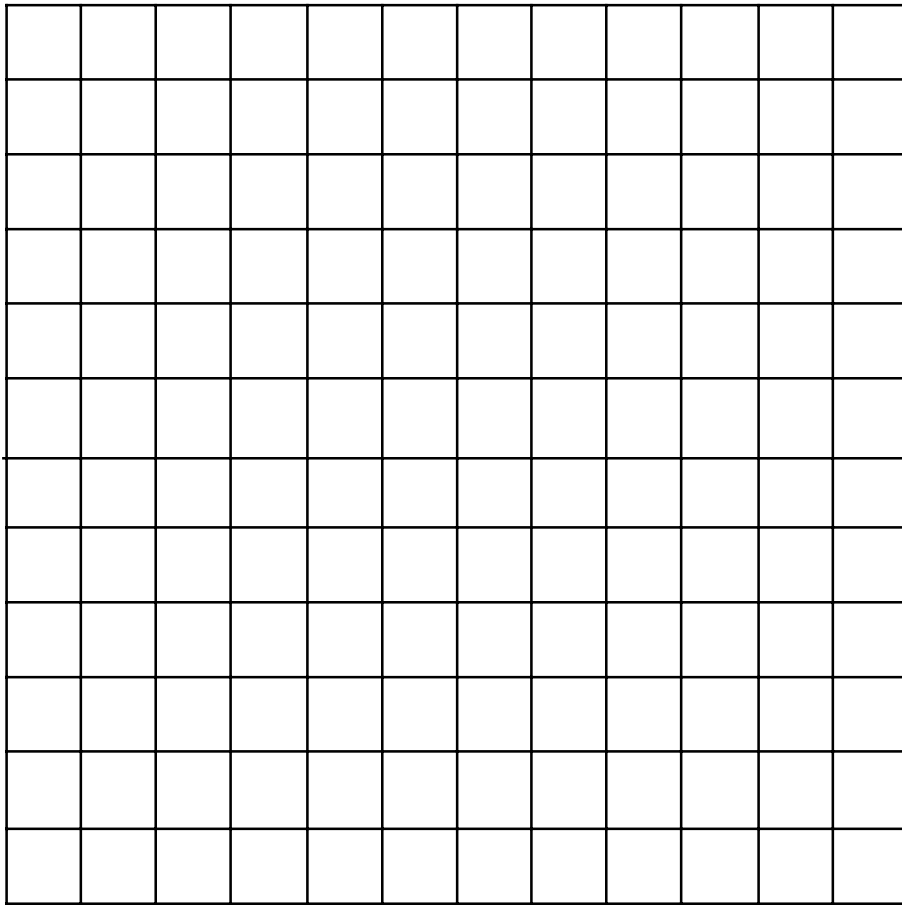


height

2. Add a cup to the stack by placing it inside the first cup. Now find the new height of the stack and record it on the table.
3. Add another cup to the stack and find your new height. Now repeat for the fourth cup.
Predict the height of 5 cups _____ 10 cups _____.

Number of Cups Added	Height of Stack (cm)
0	_____ (initial height)
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

4. *Using the grid below, display the data in a scatter plot.*



5. What is the relationship between the number of cups added to the stack and the height?

6. What is the initial height of the stack? _____

7. What change (pattern) did you observe as you added cups to the stack?

8. Was this change constant or did it vary? _____
Explain.

9. If x represents the number of cups added to the initial cup and y represents the height of the cups, what equation represents the total height of the stack? Write the equation on the line below.

HINT:

Total height	=	initial height	+	# of cups added times increase per cup
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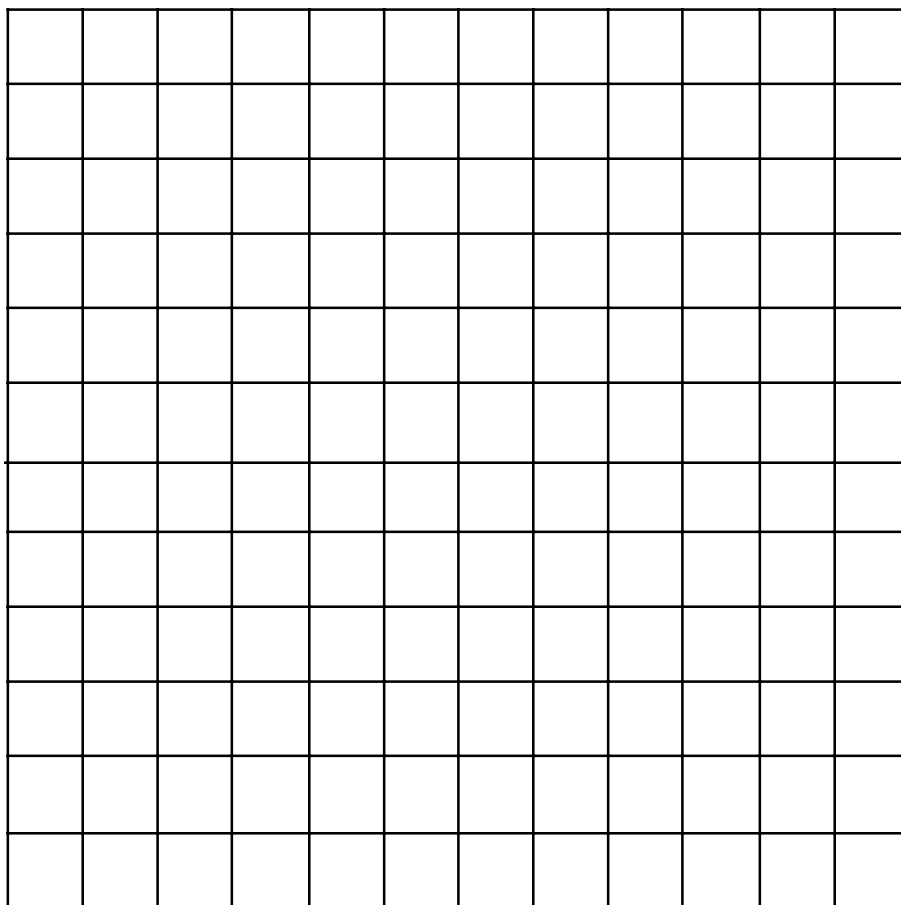
PART B

Using the larger styrofoam cups, work with your partner to find the height of 1 cup (initial height).

1. Create a data table in the space below for the measurement of the height as the number of cups increases. Complete the table for a stack of 6 cups.

Number of Cups Added	Height of Stack (cm)
0	_____ (initial height)
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

2. Using the grid below, display the data in a scatter plot .



3. If x represents the number of cups added to the initial cup and y represents the height of the cups, what equation represents the total height of the stack ? Write the equation on the line below.

4. Your parents ordered styrofoam cups from a new company. When you unpacked the box you realized that they are not the same sizes as the old cups. What could you do to find the best number of cups per stack?

“Clean Up” Performance Assessment

At the end of each day, you must mop the floor in the coffee shop. In order to do this, you need to stack the chairs so that they take up only a small portion of the floor. There are 102 chairs you must stack and the height of the ceiling in the coffee shop is 8 feet. Your job is to decide how many stacks of chairs you will need to have and how many chairs will be in each stack. To do this, complete the following activity. You will complete Part A with your group and Part B by yourself.

Part A: (with your group)

- 1. Measure (in inches) the height of the beginning chair. _____*
- 2. Add one chair and measure the height of the two chairs. _____*
- 3. Add one chair and measure the height of the three chairs. _____*

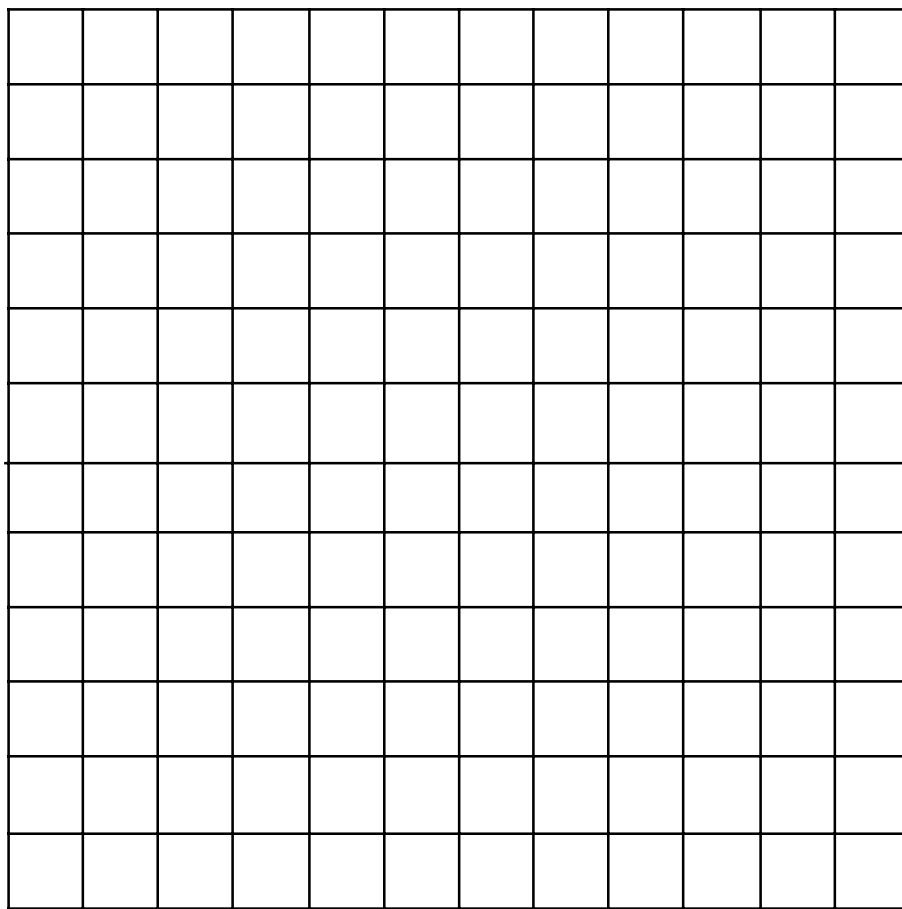
Part B: (by yourself)

- 4. Complete the following table of values.*

Number of Chairs Added	Height of Chairs (inches)
0	_____ (height of one chair)
1	_____
2	_____
3	_____
4	_____
5	_____
10	_____
120	_____

- 5. Write an equation that represents the relationship between the number of chairs and the height of the stack. Remember to define your variables.*

6. Use the grid below to construct a scatter plot that represents the number of chairs added and the height of the chairs.



7. If you were to put all 120 chairs in one stack, how high would the stack be? _____

8. Would this be a reasonable thing to do? Explain why or why not.

9. Using the data from your table or graph, determine how many chairs you would put in each stack and how many stacks you would have. Explain why and how you arrived at your answer. Use mathematical reasoning to justify your answer.
